98 學年度台灣聯合大學系統學士班轉學考選擇題正確答案

考科名稱：微積分

類組別：A2、A3、A4、A5、A6, 026

1	E	2	E	3	A	4	D	5	B
6	D	7	A	8	B	9	C	10	D
11		12		13		14		15	
16		17		18		19		20	
21		22		23		24		25	
26		27		28		29		30	
31		32		33		34		35	
36		37		38		39		40	
41		42		43		44		45	
46		47		48		49		50	

甲、選擇題：共 10 題，每題 4 分，共 40 分。請用大寫字母 A, B, C, D 或 E 答題，並將答案依題號順序寫在答案卷上。皆單選。

1. What is the maximum value $|f''(x)|$ for the function $f(x) = x^3(10-3x^2)$ on the closed interval $[0,2]$?

(A) 16 (B) 660 (C) $\frac{40}{\sqrt{3}}$ (D) $20\sqrt{3}$ (E) 360

2. If $\int_0^{x^2} f(t)\,dt = x\sin\pi x$, what is the value of $f(4)$?

(A) 0 (B) -2π (C) 2π (D) $-\frac{\pi}{2}$ (E) $\frac{\pi}{2}$

3. What is the value of $\lim_{n\to\infty}\int_0^1 \frac{ny^{n-1}}{1+y}\,dy$?

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 0 (D) $+\infty$ (E) $-\infty$

4. Which of the following series *diverges*?

(A) $\sum_{n=1}^{\infty}\frac{\ln n}{n^{3/2}}$ (B) $\sum_{n=1}^{\infty}\frac{\tan^{-1} n}{n^2+1}$ (C) $\sum_{n=1}^{\infty}\frac{2^n 3^n}{n^n}$ (D) $\sum_{n=1}^{\infty}(-1)^n(\sqrt{n^2+n}-n)$ (E) $\sum_{n=1}^{\infty}\sin\frac{1}{n^2}$

5. What is the value of the definite integral $\int_0^1 \frac{x^3 e^{x^2}}{(x^2+1)^2}\,dx$?

(A) $\frac{2e-5}{8}$ (B) $\frac{e-2}{4}$ (C) $\frac{e-1}{2}$ (D) $e-1$ (E) None of the above

6. Which function has maclaurin series $\sum_{n=0}^{\infty}(-1)^n 2^n x^n$?

(A) $\tan^{-1}2x$ (B) $\ln(1+2x)$ (C) $\ln\left(\frac{1+x}{1-x}\right)$ (D) $\frac{1}{1+2x}$ (E) $\sin^{-1}2x$

7. What is the area that lies inside the circle $r=6$ and above the line $r = 3\csc\theta$?

(A) $12\pi - 9\sqrt{3}$ (B) $36\pi - 9\sqrt{3}$ (C) $12\pi + \sqrt{3}$ (D) $12\pi - \sqrt{3}$ (E) $36\pi - \sqrt{3}$

8. What is the value of the double integral $\int_0^{\ln 10}\int_{e^x}^{10}\frac{1}{\ln y}\,dy\,dx$?

(A) ln 9 (B) 9 (C) 10 (D) ln 10 (E) None of the above

9. Which of the following curve is tangent to the surface $x^2+y^2-z=1$ when $t=1$?

(A) $\mathbf{r}(t) = (2t-1)\mathbf{i} + \sqrt{t}\,\mathbf{j} + \sqrt{t}\,\mathbf{k}$ (B) $\mathbf{r}(t) = \sqrt{t}\,\mathbf{i} + (2t-1)\mathbf{j} + \sqrt{t}\,\mathbf{k}$
(C) $\mathbf{r}(t) = \sqrt{t}\,\mathbf{i} + \sqrt{t}\,\mathbf{j} + (2t-1)\,\mathbf{k}$ (D) $\mathbf{r}(t) = t\,\mathbf{i} + t\,\mathbf{j} + \sqrt{t}\,\mathbf{k}$ (E) None of the above

10. Let S be the surface parametrized by $\mathbf{r}(\theta, z) = 3\sin 2\theta\,\mathbf{i} + 6\sin^2\theta\,\mathbf{j} + z\,\mathbf{k}$ where $0 \le \theta \le \pi$. Which of the following is equivalent to the surface differential $d\sigma$?

(A) $\frac{1}{3}\,d\theta dz$ (B) $d\theta dz$ (C) $3\,d\theta dz$ (D) $6\,d\theta dz$ (E) $12\,d\theta dz$

參考用

注意：背面有試題

乙、填充題：共 5 題，每題 6 分，共 30 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Find the limit: $\lim_{x\to 0^+}\left(\frac{\sin x}{x}\right)^{1/x^2}$. Answer : ________

2. Among all the points on the graph of parabolid $z = 10 - x^2 - y^2$ that above the plane $x + 2y + 3z = 0$, find the point farthest from the plane. Answer : ________

3. Set up the integral with the order $dzdrd\theta$ for evaluating the triple integral of the function $F(x, y, z) = \sin(z^2)$ over the solid region D bounded below by the plane $z = 0$, laterally by the circular cylinder $(x - 1)^2 + y^2 = 1$, and above by the paraboloid $z = 2 - x^2 - y^2$. Answer : ________ **(Do not evaluate the integral.)**

4. Use the transformation (chang of variables) $x = uv^{-1}$, $y = uv$ to evaluate

$$\iint_D (x^2 + y^2)\, dxdy$$

where $D = \{(x, y) : 1 \le xy \le 4,\ 1 \le y/x \le 4\}$. Answer : ________

5. At the point (1,2), the function $f(x, y)$ has a derivative of 2 in the direction toward (2,2) and a derivative of -2 in the direction toward (1,1). Find the derivative of f at (1,2) in the direction toward the point (4,6). Answer : ________

丙、計算、證明題：共 3 題，每題 10 分，共 30 分。須詳細寫出演算過程，否則不予計分。

1. Define $f(x) = \begin{cases} x^3 \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$ Show that $f'(x)$ is continuous at $x = 0$.

2. Find the values $a \le b$ such that $\int_a^b 1 - e^{(1-x^2)}\, dx$ has minimal value.

3. Let C be a simple colsed smooth curve in the plane $x + 2y + 2z = 2$. (Orient C to be counterclockwise when view from above.) Show that

$$\oint_C 3y\, dx - 2z\, dy + x\, dz$$

depends only on the area of the region enclosed by C and not on the position or shape of C.

參考用